Appendix C

A person's height can be measured in units such as feet, inches, or meters. Conversion factors allow you to convert from one unit to another.

### Conversion Problems and Dimensional Analysis

Many problems in both everyday life and in the sciences involve converting measurements. These problems may be simple conversions between the same kinds of measurement. For example:

**a.** A person is five and one-half feet tall. Express this height in inches.

**b.** A flask holds 0.575 L of water. How many milliliters of water is this?

In other cases, you may need to convert between different kinds of measurements.

**c.** How many gallons of gasoline can you buy for $15.00 if gasoline costs $1.42/gallon?

**d.** What is the mass of 254 cm³ of gold if the density of gold is 19.3 g/cm³?

More complex conversion problems may require conversions between measurements expressed as ratios of units. Consider the following examples.

**e.** A car is traveling at 65 miles/hour. What is the speed of the car expressed in feet/second?

**f.** The density of nitrogen gas is 1.17 g/L. What is the density of nitrogen expressed in micrograms/deciliter (µg/dL)?

Problems a through f can be solved using a method that is known by a few different names—dimensional analysis, factor label, and unit conversion. These names emphasize the fact that the dimensions, labels, or units of the measurements in a problem—the units in the given measurement(s) as well as the units desired in the answer—can help you write the solution to the problem.

Dimensional analysis makes use of ratios called conversion factors. A conversion factor is a ratio of two quantities equal to one another. For example, to work out problem a, you must know the relationship 1 ft = 12 in. The two conversion factors derived from this equality are shown below.

\[
\frac{1 \text{ ft}}{12 \text{ in}} = 1 \text{(unity)} = \frac{12 \text{ in}}{1 \text{ ft}}
\]

To solve problem a by dimensional analysis, you must multiply the given measurement (5.5 ft) by a conversion factor that allows the feet units to cancel, leaving the unit inches—the unit of the requested answer.

\[
5.5 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} = 66 \text{ in}
\]
Carefully study the solutions to the remaining five example problems below. Notice that in each solution, the conversion factors are written so that the unit of the given measurement cancels, leaving the correct unit for each answer. When working conversion problems, the equalities needed to write the conversion factor may be given in the problem. This is true in examples c and d. In other problems, you need to either know or look up the necessary equalities, as in examples b, e, and f.

b. 0.575 L × \(\frac{10^3 \text{ mL}}{1 \text{ L}}\) = 575 mL.

c. $15.00 \times \frac{1 \text{ gal}}{\$1} = 10.6 \text{ gal}$

d. 254 cm \(\times\) \(\frac{1 \text{ in}}{2.54 \text{ cm}}\) = 9.96 \text{ in}

e. \(65 \text{ mi/h} \times \frac{1 \text{ mi}}{1 \text{ h}}\) \(\times\) \(\frac{1 \text{ h}}{3600 \text{ s}}\) = 18.6 ft/s

f. \(\frac{1.17 \text{ g}}{1 \text{ L}} \times \frac{10^6 \text{ mg}}{1 \text{ g}} \times \frac{1 \text{ L}}{10 \text{ dL}}\) = 1.17 \text{ mg/dL}

**SAMPLE PROBLEM MH-5**

**Applying Dimensional Analysis**

A grocer is selling oranges at “3 for $1.” How much would it cost to buy a dozen oranges?

**Solution**

The following equality is given in the problem.

3 oranges = $1

You can write two conversion factors based on this relationship.

\[ \frac{\$1}{3 \text{ oranges}} \text{ and } \frac{3 \text{ oranges}}{\$1} \]

The given unit is oranges; the desired unit is dollars. Thus, use the conversion factor on the left to convert from oranges to dollars. One dozen equals 12, so you can start the calculation with the measurement 12 oranges.

\[ 12 \text{ oranges} \times \frac{\$1}{3 \text{ oranges}} = \$4 \]

The given unit (oranges) cancels, leaving the desired unit (dollars) in the answer.
Practice the Math

Use the following equalities for Questions 1–3.

- 60 s = 1 min
- 12 in = 1 ft
- 60 min = 1 h
- 3 ft = 1 yd
- 24 h = 1 day
- 1 min 5.50 yd = 1 rod
- 7 days = 1 wk
- 5280 ft = 1 mi
- 365 days = 1 yr
- 12 in = 1 ft
- 1 ft 7 days = 1 wk
- 201x850 /H11005 1 h
- 5280 ft = 1 mi
- 365 days = 1 yr
- 3 ft = 1 yd
- 365 days = 1 yr
- 24 h = 1 day

1. Write the conversion factor need for each unit conversion.
   a. feet → yards
   b. yards → rods
   c. yards → miles
   d. feet → seconds
   e. feet → miles
   f. seconds → minutes

2. Solve each problem by dimensional analysis.
   a. How many feet long is the 440-yard dash?
   b. Calculate the number of minutes in two weeks.
   c. Calculate the number of days in 1800 h.
   d. How many miles is 660 ft?
   e. How many inches long is a 100-yd football field?
   f. Calculate the number of hours in one year.
   g. How many rods are in 12 miles?
   h. Calculate the number of minutes in 7 days.

3. Solve each problem by dimensional analysis.
   a. A student walks at a brisk 3.50 mi/h. Calculate the student’s speed in yards/minute.
   b. Water runs through a hose at the rate of 2.5 gal/min. What is the rate of water flow in units gallons/day?
   c. A clock gains 2.60 s each hour (2.6 s gained/h). What is the rate of time gained in minutes/week?
   d. A spider travels 115 inches in 1 min (speed = 115 in/min). What is the speed of the spider in miles/hour?

Applying Dimensional Analysis to Chemistry

Use the following metric relationships to work out Questions 4 and 5.

- 10^3 m = 1 km
- 10 dm = 1 m
- 10 cm = 1 m
- 10^3 mm = 1 m
- 10^6 μm = 1 m
- 1 g H_2O = 1 mL H_2O

4. Perform the following conversions.
   a. 45 m to kilometers
   b. 4 × 10^7 nm to meters
   c. 8.5 dm to millimeters
   d. 8.2 × 10^4 μm to centimeters
   e. 0.23 km to decimeters
   f. 865 cm^2 to liters
   g. 7.28 × 10^7 pm to micrometers
   h. 56 g H_2O to L H_2O

5. Perform the following conversions.
   a. 4.5 m/s to millimeters/minute
   b. 7.9 × 10^3 km/h to decimeters/minute
   c. 77 mL H_2O/s to liters H_2O/hour
   d. 3.34 × 10^6 nm/min to centimeters/second